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Total No. of Questions : 09]

[Total No. of Pages : 04

B.Tech. (Sem. - 1st / 2nd)

ENGINEERING MATHEMATICS - II

SUBJECT CODE : AM - 102 (2K9 Batch)

Paper ID : [A0119]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Five** questions from Section - B and C.
3) Select atleast **Two** questions from Section - B and C.

Section - A

Q1) *(Marks : 2 each)*

a) Let T be a transformation defined from R^1 into R^3 defined by

$$T(x) = (x, x^2, x^3)$$

Is T linear or not?

b) For what value (s) of k do the set of vectors $\{(k, 1, 1), (0, 1, 1), (k, 0, k)\}$ in R^3 are Linearly independent.

c) Examine whether the matrix A is similar to B, where

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ and } B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

d) Check the equation

$$(3x^2 + 2e^y)\, dx + (2x\, e^y + 3y^2)\, dy = 0$$

for exactness.

e) Give the physical interpretation of gradient of a scalar point function.

f) If $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that curl $(\vec{a} \times \vec{r}) = 2\vec{a}$.

g) Find the work done by the force field

$$\vec{F} = -xy\,\hat{i} + y^2\,\hat{j} + z\,\hat{k}$$

in moving a particle over the circular path $x^2 + y^2 = 4$, $z = 0$ from (2, 0, 0) to (0, 2, 0).

h) A problem in physics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?

i) Mention any four chief characteristics of Normal Probability Curve.

j) Define Type - I and Type - II errors used in Testing of Hypothesis.

Section - B

(Marks : 8 each)

Q2) (a) Find the real value λ for which the system of equations:

$$x + 2y + 3z = \lambda x,\ 3x + y + 2z = \lambda y,\ 2x + 3y + z = \lambda z$$

have non-trivial solution.

(b) State Cayley - Hamilton theorem and use it to find A^8, given that

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}.$$

(c) If λ is an eigen value of the matrix A, then prove that $g(\lambda)$ is an eigen value of $g(A)$, where g is a polynomial.

Q3) (a) Find the general solution of the equation $3x^4 p^2 - xp - y = 0$, where $p = \frac{dy}{dx}$.

(b) Solve the differential equation:

$$(x^2 y^2 + xy + 1)\, ydx + (x^2 y^2 - xy + 1)\, xdy = 0.$$

Q4) (a) Use method of variation of parameters to solve the equation:

$$y'' - 6y' + 9y = \frac{e^{3x}}{x^2}.$$

(b) Solve the simultaneous differential equations:

$$\frac{dx}{dt} + 5x - 2y = t,\ \frac{dy}{dt} + 2x + y = 0.$$

Q5) The voltage V and the current i at a distance x from the sending end of the transmission line satisfy the equations:

$$\frac{-dv}{dx} = Ai, \quad \frac{-di}{dx} = BV,$$

where A and B are constants. If $V = V_0$ at the sending end ($x = 0$) and $V = 0$ at the receiving end ($x = l$), then show that

$$V = V_0 \left\{ \frac{\sin h \; n(1-x)}{\sin h \; n \; l} \right\}, \text{ where } n^2 = AB.$$

Section - C

(Marks : 8 each)

Q6) (a) Prove the identity

$$\nabla \times \left(\vec{F} \times \vec{G}\right) = \vec{F}\left(\nabla \cdot \vec{G}\right) - G\left(\nabla \cdot \vec{F}\right).$$

(b) Evaluate $\iint_S \vec{F} \cdot \hat{n}\, ds$, where

$\vec{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the surface $2x + 3y + 6z = 12$ in the first Octant.

Q7) (a) State Stoke's theorem and use it to evaluate the line integral.

$$\oint_C \vec{F} \cdot d\vec{r},$$

where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0).

(b) A vector field is given by

$$\vec{F} = \left(x^2 - y^2 + x\right)\hat{i} - (2xy + y)\hat{j}$$

Show that the field is irrotational and find its scalar potential.

Q8) (a) The probability of a man hitting a target is $\frac{1}{4}$. If he fires 7 times, then what is the probability of his hitting the target at least twice. Also find that how many times must he fire so that the probability of his hitting the target at least once is greater than $\frac{2}{3}$.

(b) Explain the method of least squares to fit a curve and use it to fit a straight line $y = a + bx$ to the data:

x:	2	7	9	1	5	12
y:	13	21	23	14	15	21

Q9) (a) To compare two kinds of bumper guards, 6 of each kind were mounted on a certain kind of compact car. Then each car was run into a concrete wall at 5 miles per hour, and the following are the costs of the repairs (in dollars):

Bumper Guard I :	107	148	123	165	102	119
Bumper Guard II :	134	115	112	151	133	129

use the 0.01 level of significance to test whether the difference between the two sample means is significant. Given that

$t_{0.005}$ for 10 $d.f = 3.169$, $t_{0.01}$ for 10 $d.f = 2.764$,

$t_{0.005}$ for 12 $d.f = 3.055$, $t_{0.01}$ for 12 $d.f = 2.681$

(b) In one sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in other sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level of significance. Given that $F_{0.05}$ for (7, 9) $d.f = 3.29$ and $F_{0.05}$ for (9, 7) $d.f = 3.68$.